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SOME NOTES ON CONSTRUCTION AND APPLICATION

OF

POISSON OPERATING CHARACTERISTIC FUNCTIONS

FOR

EQUIPMENT MTBF DECISION-MAKING

George H. Allen

MARCH 1965

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FOREWORD

During 1963 - 1964 period, the ESD Reliability/Maintainability staff conducted a series of lectures and briefings on R/M. Several lectures dealt with equipment MTBF demonstration problems.

This TDR summarizes a discussion conducted on the applications of the Cumulative Poisson Function to MTBF demonstration. The ideas produced during this and subsequent discussions are reflected in ESD-TDR-64-616.

REVIEW AND APPROVAL

This technical report has been reviewed and is approved.



FRANK E. BRANDEBERRY

Colonel, USAF

Chief, Technical Requirements & Standards

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SECTION I

INTRODUCTION

The specification of a minimum MTBF for an equipment introduces the problem of demonstration. At ESD, we are usually operating with at least four constraints:

1. Equipment MTBFs are in the order of 500 hours or more.
2. Limited numbers (usually one or two) of articles are purchased.
3. Delivery schedules are compressed.
4. A finite number of dollars are available.

With such constraints, the present reliability demonstration model defined in MIL-R-26474 does not offer a satisfactory solution to the demonstration problem. This model could suspend decisions on compliance to contractual MTBFs for as much as a year (some of our programs require delivery within 8 to 10 months from contract award). The result, when we attempt to apply and enforce MIL-R-26474 demonstration criteria, is usually a request for waiver on the part of a contractor and the development of "home-made" decision criteria as to the acceptability of equipment. These criteria are usually non-quantitative and involve personal expressions; such as, "I think we should accept (or reject)". What is needed, of course, is a quantitative decision rule or model which has at least the following essential properties:

1. It is understood by both the customer (ESD) and contractor.
2. Quantifies the risks involved in decisions; i.e., the probability of rejecting equipments which have achieved the contractual MTBF and the probability of accepting equipments which have not achieved the contractual MTBF.
3. Allows scheduling and cost analysis within the overall constraints of the program.

The ESD is beginning to apply a simple decision rule which we feel satisfies the above essential properties. This rule states that equipment will be planned to be exercised for a specific amount of time (such as one multiple of contractual MTBF) and during this time a specific number of failures will be allowed. If this number is exceeded, the equipment will be declared unacceptable.

For example, take the case where test time is set equal to θ_c , the contractual MTBF. If c is set equal to zero, a contractor would have a 39% chance that, even if his equipment had an MTBF equal to $2\theta_c$, the equipment would be rejected. On the other hand, the customer has a 14% chance of accepting an equipment which has an MTBF equal to $1/2 \theta_c$. Such a rule, while protecting the customer, would probably be unacceptable to a contractor. To minimize his risk, c could be set equal to one. If his equipment had an MTBF equal to $2 \theta_c$, his risk (probability of rejection) would now be 8%, but the customer would have a 42% chance of accepting an equipment which has an MTBF equal to $1/2 \theta_c$.

This simple arithmetic illustrates the need for clearly setting forth and obtaining agreement on a value of c . We expect that some discussion on this matter will take place in technical proposals and in contract negotiations. The point of consideration in these discussions is the fact that both parties are aware of each others' risks.

If by accident, ESD cites MIL-R-26474 techniques in an RFP, we expect bidders to take us to task when they discover that such techniques are unrealistic for the program under consideration. We also hope that they will consider and offer suggestions for a fixed test time, allowable number of failures, type of demonstration (or another suitable technique).

SECTION II

THE CUMMULATIVE POISSON MODEL

The selection of an allowable number of failures (c) is a matter of concern not only for ESD but also a contractor. In the first place, as indicated in Section I, c is dependent on the number of programmed test hours. Usually, the greater the test time, the greater the value taken for c . This results from the consideration that as c increases, the probability of acceptance increases and, therefore, the contractor's or producer's statistical risk decreases.

However, for a given c value and test time, a contractor can reduce his statistical risk by increasing the value of the ratio of "true" MTBF capability to contractual MTBF, (θ_t/θ_c) . Estimates of θ_t are provided during the design of an equipment in terms of MTBF predictions.

These few statistical concepts can be illustrated with a family of Poisson operating characteristic functions. A purpose of this discussion is to indicate a method by which such functions can be constructed.

The Cumulative Poisson expression for c or less failures in t hours of operation, given an MTBF equal to θ_t , is

$$\begin{aligned} \text{EQ.1} \quad P(x \leq c) &= \sum_{x=0}^{x=c} e^{-t/\theta_t} (t/\theta_t)^x / x! \\ x &= 0, 1, 2, \dots \end{aligned}$$

To illustrate the application of the above equation, assume $\theta_t = 1000$ hours and that it has been decided to test for one multiple of θ_t . The probability of exactly zero failures during the test is,

$$\begin{aligned} \text{EQ.2} \quad P(x = 0) &= e^{-1000/1000} (1000/1000)^0 / 0! \\ &= 0.37 \end{aligned}$$

The probability of one or less failures during the test is,

$$\begin{aligned} \text{EQ.3} \quad P(x \leq 1) &= \sum_{x=0}^{x=1} e^{-1000/1000} (1000/1000)^x / x! \\ &= 0.74 \end{aligned}$$

If the MTBF is 2000 hours, then, during a 1000 hour test, the probability of one or less failures is 0.92. If, on the other hand, the MTBF is 500 hours, the probability of one or less failures is 0.42.

The practical significance of the above numbers is interesting. Assume that the demonstration decision-rule for a 1000 hour MTBF stated: "test for 1000 hours of satisfactory operation with no more than one failure". A contractor would have a 0.08 statistical chance that even if his equipment had a 2000 hour MTBF capability it would be rejected (experience two or more failures) by the decision-rule. On the other hand, with an MTBF capability of 500 hours (1/2 the MTBF requirement) a contractor has a 0.42 statistical probability that his equipment still will be accepted by the decision-rule.

If the latter statistical probability is disturbing to ESD, a reduction in the permissible number of failures from $c = 1$ to $c = 0$ would allow only a 0.14 probability that equipment with 1/2 the MTBF requirement would be accepted. On the other hand, to have only a 0.10 probability of rejection, a contractor's equipment would have to have a 10,000 hour MTBF capability. To design this capability may be either technically impossible or economically unattractive to both ESD and a contractor and a further analysis of the demonstration decision-rule would be required.

This arithmetic illustrates an approach to the development of Poisson operating characteristic functions. Such functions relate the probability of acceptance to the ratio (θ_t/θ_c) for values of c .

SECTION III

SOME SAMPLE POISSON OPERATING CHARACTERISTIC FUNCTIONS

Table 1 presents values of three Poisson operating characteristic functions when test time is set equal to one multiple of contractual MTBF. Figure 1 is a graph of these three functions. As the functions indicate, the most favorable decision-rule, of the three presented for ESD, occurs when c is set equal to zero. On the other hand, as expected, the most favorable decision-rule to a contractor occurs when c is set equal to two.

The reasoning behind the above observations is apparent from the probability of acceptance values in Table 1. ESD is concerned with the probability of accepting equipment which is "unsatisfactory", i.e., probability of accepting when $\theta_t/\theta_c \leq 1$. These probabilities decrease either as the ratio θ_t/θ_c decreases and/or the value of c decreases. For example: if one failure is permitted in a test of one multiple of MTBF duration, the probability of accepting equipment which has $1/2$ the MTBF requirement is 0.42; when c is set equal to zero, this probability decreases to 0.14. The probabilities of accepting equipment which has $1/4$ the MTBF requirement when $c = 1$ and $c = 0$ are 0.10 and 0.02, respectively.

A compromise decision-rule, as indicated in Figure 1, would involve setting $c = 1$. This means that the probability of rejection for all values of $\theta_t/\theta_c \geq 1$ never exceeds 0.26. When the ratio $\theta_t/\theta_c = 2$, there is only a 0.08 chance that two or more failures will occur. On the other hand, there is still a 0.10 statistical probability that, when $\theta_t/\theta_c = 1/4$, one or less failures will occur.

If it were possible to extend test duration to, say, two multiples of contractual MTBF, a different family of Poisson operating characteristic functions could be generated. Some values of these functions are given in Table 2. A graphical presentation is given in Figure 2. Table 1 and 2 values indicate the effect on statistical probability of acceptance when test duration is increased and c is held constant. For example, if $c = 1$, the statistical probability of accepting equipment which has an MTBF equal to $1/2$ the contractual value is 0.42, for the case when test duration is equal to one multiple of required MTBF, but is only 0.10, when test duration is two times the contractual value.

Additional computations can be performed for different values of test duration. To illustrate the ease by which this may be accomplished, a detailed example is presented in Section V for the case when test duration is equal to three multiples of contractual MTBF and $\theta_t/\theta_c = 2$.

Table 1

Some Values of Poisson Operating Characteristic Functions
When Test Duration Equals The Contractual MTBF Value

<u>θ_t/θ_c</u>	<u>$c = 0$</u>	<u>$c = 1$</u>	<u>$c = 2$</u>
1/4	0.02	0.10	0.26
1/2	0.14	0.42	0.70
1	0.37	0.74	0.87
3/2	0.51	0.85	0.97
2	0.61	0.92	1

Figure 1

Some Poisson Operating Characteristic Functions
When Test Duration Equals One Multiple of MTBF (θ_c)

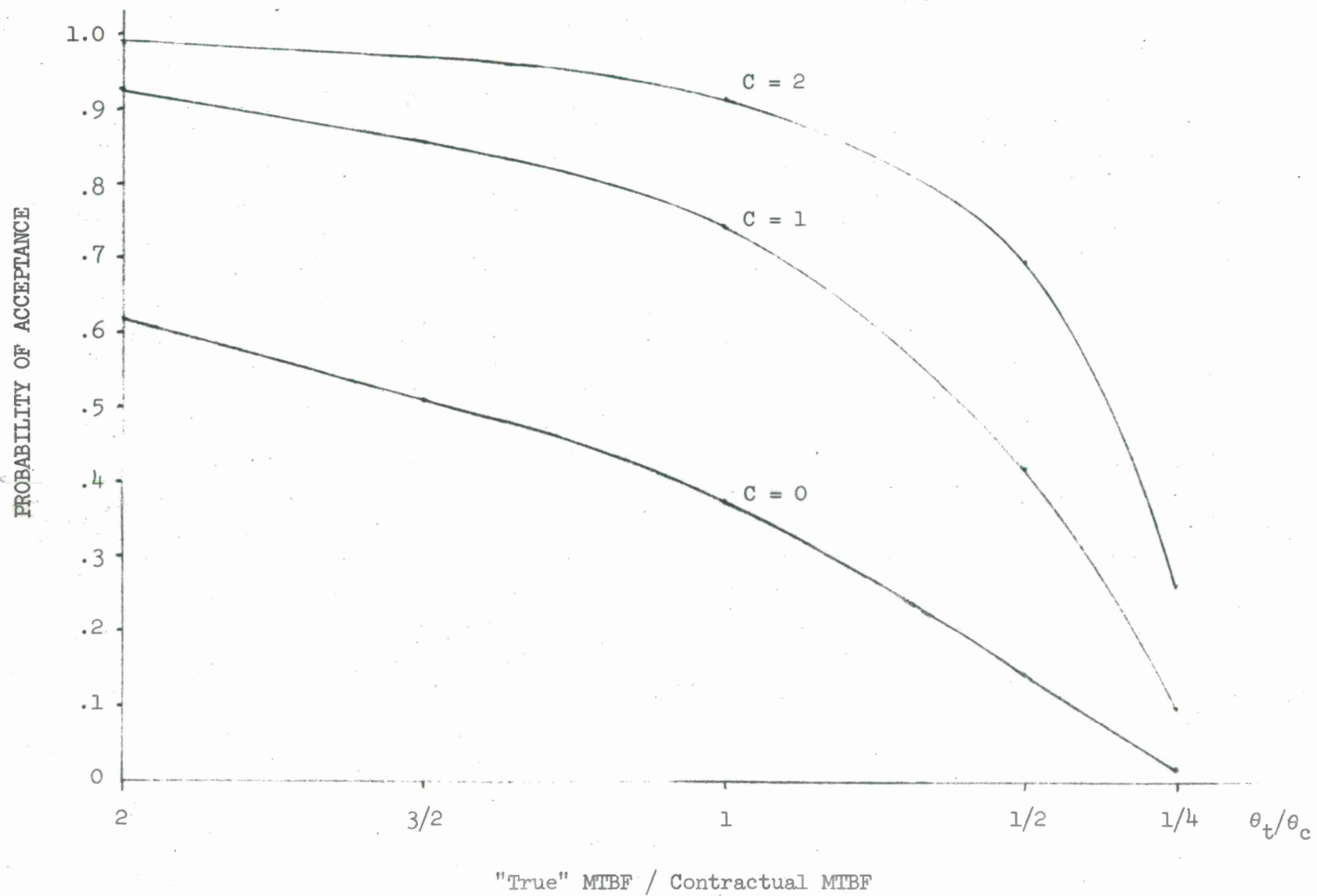


Table 2

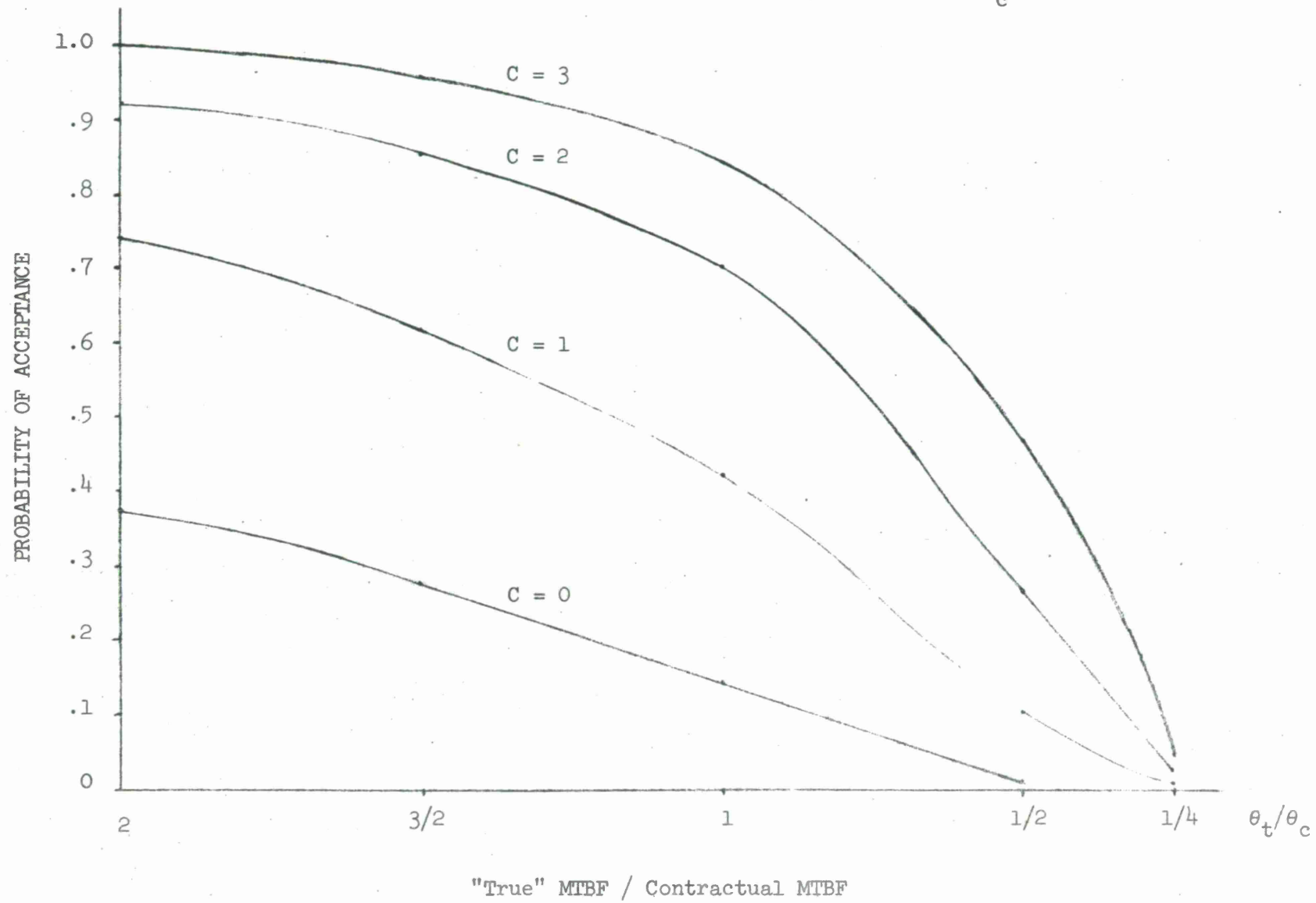
Some Values of Poisson Operating Characteristic Functions
When Test Duration Equals Twice The Contractual MTBF Value

<u>θ_t/θ_c</u>	<u>$c = 0$</u>	<u>$c = 1$</u>	<u>$c = 2$</u>	<u>$c = 3$</u>
1/4	0.0003	0.0024	0.01	0.04
1/2	0.02	0.10	0.26	0.47
1	0.14	0.42	0.70	0.89
3/2	0.27	0.62	0.85	0.95
2	0.37	0.74	0.92	0.98

∞

Figure 2

Some Poisson Operating Characteristic Functions
When Test Duration Equals Two Multiples of MTBF (θ_c)



SECTION IV

SUMMARY

To assist in the selection of a c value for a demonstration problem which satisfies the constraints discussed in Section I, it is suggested that Poisson operating characteristic functions be developed. Such functions present, for selected c and test duration values, the probabilities of accepting equipment which has either met, failed to meet, or exceeded an MTBF requirement.

For ease of computational presentation, only integral multiples of contractual MTBF have been used in this discussion in assigning a test duration value. Obviously, in a given demonstration problem, other numbers can be used. A selection of a test duration value depends on such considerations as program schedules, quantities of equipment available for test, availability of test dollars, etc.

It is recognized that the Cumulative Poisson Function is only one statistical model presently available for application on equipment MTBF demonstration problems and that the Poisson approach, with test duration held to a minimum, does not give a high statistical confidence that the contractual MTBF has been achieved or exceeded. But, with the constraints of Section I, it at least satisfies the need for a quantitative decision-rule. Furthermore, from the reliability engineering management viewpoint, a one or two multiple of MTBF test with, say, no more than one failure is not considered to be a unsatisfactory procedure. Rather, it could cause contractor management to note the need for delivering equipment for demonstration which has been designed and manufactured for reliable operation, especially, when supplemented by a retest penalty clause which places the complete cost burden of failing the original demonstration on a contractor.

Furthermore, the budgeting problem for a fixed test time reliability demonstration is easier to solve than one influenced by a variable test time.

The statistical ideas in this discussion can be extended to establish a more comprehensive view of the reliability demonstration problem. A task to accomplish this objective has been undertaken by the Technical Requirements and Standards Office.

SECTION V

SAMPLE COMPUTATION

$$P(c) = \frac{e^{-t/\theta_t} (t/\theta_t)^c}{c!}$$

Assume the test duration is to be fixed at three times the contractual MTBF; i.e., $3\theta_c$. If θ_t is two times the contractual MTBF,

$$\begin{aligned} P(c) &= \frac{e^{-3\theta_c/\theta_t} (3\theta_c/2\theta_c)^c}{c!} \\ &= \frac{e^{-1.5} (1.5)^c}{c!} \end{aligned}$$

$$P(0) = \frac{e^{-1.5}}{0!} = 0.22$$

$$P(1) = \frac{e^{-1.5} (1.5)^1}{1!} = 0.33$$

$$P(2) = \frac{e^{-1.5} (1.5)^2}{2!} = 0.25$$

$$P(3) = \frac{e^{-1.5} (1.5)^3}{3!} = 0.12$$

Therefore,

$$P(1 \text{ or less}) = 0.22 + 0.33 = 0.55$$

$$P(2 \text{ or less}) = 0.55 + 0.25 = 0.80$$

$$P(3 \text{ or less}) = 0.80 + 0.12 = 0.92$$

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